Lecture 3
Blackbody radiation. Main Laws.

Objectives:

1. Concepts of a blackbody, thermodynamical equilibrium, and local thermodynamical equilibrium.

2. Main laws:

   - Planck function.
   - Stefan-Boltzmann law.
   - Wien's displacement law.
   - Kirchhoff's law.

3. Emissivity of the surfaces and the atmosphere.

Required reading:

L02: 1.2, 1.4.3

Additional/advanced reading:

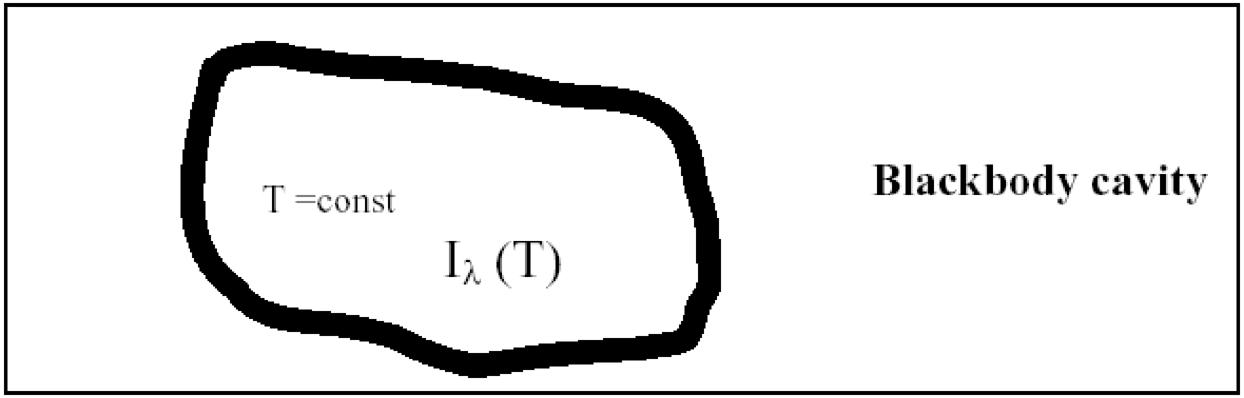
L02: Appendix A;

T&S99: pp.96-97

1. Concepts of a blackbody and thermodynamical equilibrium.

Thermodynamical equilibrium describes the state of matter and radiation inside an isolated constant-temperature enclosure.

Blackbody radiation is the radiative field inside a cavity in thermodynamic equilibrium.



Emission of Radiation

Emission is the creation of radiation by matter. From quantum mechanics: atoms or molecules in excited states decay and the energy is converted to photons.

In thermodynamic equilibrium there is both thermal equilibrium and radiative equilibrium (emitted radiation equals absorbed radiation). This gives a particular distribution of excited states, and hence a particular radiation spectrum.

A *blackbody cavity* is a closed system in thermodynamic equilibrium, and has a particular emission spectrum that depends only on temperature: $I_\lambda = B_\lambda(T)$. This spectrum is called the Planck function.

It is called a blackbody because all radiation incident on cavity is absorbed. Blackbody cavities are important for calibration of IR instruments since they provide a known source of radiance.

***Properties of blackbody radiation*:**

- **Radiation emitted by a blackbody is isotropic, homogeneous and unpolarized;**
- **Blackbody radiation at a given wavelength depends only on the temperature;**
- **Any two blackbodies at the same temperature emit precisely the same radiation;**
- **A blackbody emits more radiation than any other type of an object at the same temperature;**

NOTE: The atmosphere is not strictly in the thermodynamic equilibrium because its temperature and pressure are functions of position. Therefore, it is usually subdivided into small subsystems each of which is effectively isothermal and isobaric referred to as **Local Thermodynamical Equilibrium (LTE).** (See Goody&Yung: 2.2.2)

- The concept of **LTE** plays a fundamental role in atmospheric studies: e.g., the main radiation laws discussed below, which are strictly speaking valid in **thermodynamical equilibrium**, can be applied to an atmospheric air parcel in **LTE.**

2. Main Laws

Planck function $\mathbf{B_\lambda(T)}$ gives the **intensity (or radiance)** emitted by a blackbody having a given temperature.

NOTE: Planck derived $\mathbf{B_\lambda(T)}$ by introducing photon energies, $E = \mathbf{nhc/\lambda}$

See Thomas&Stamnes or L02 (Appendix A)

- **Plank function** can be expressed in wavelength, frequency, or wavenumber domains as

$$\boxed{\begin{aligned} B_\lambda(T) &= \frac{2hc^2}{\lambda^5(\exp(hc/k_B T\lambda)-1)} \qquad [3.1] \\ B_{\tilde{\nu}}(T) &= \frac{2h\tilde{\nu}^3}{c^2(\exp(h\tilde{\nu}/k_B T)-1)} \qquad [3.2] \\ B_\nu(T) &= \frac{2h\nu^3 c^2}{\exp(h\nu c/k_B T)-1} \qquad [3.3] \end{aligned}}$$

where **λ** is the wavelength; $\tilde{\nu}$ is the frequency; **ν** is the wavenumber;

h is the Plank's constant; ***k_B*** is the Boltzmann's constant ($k_B = 1.3806\text{x}10^{-23}$ J K^{-1});

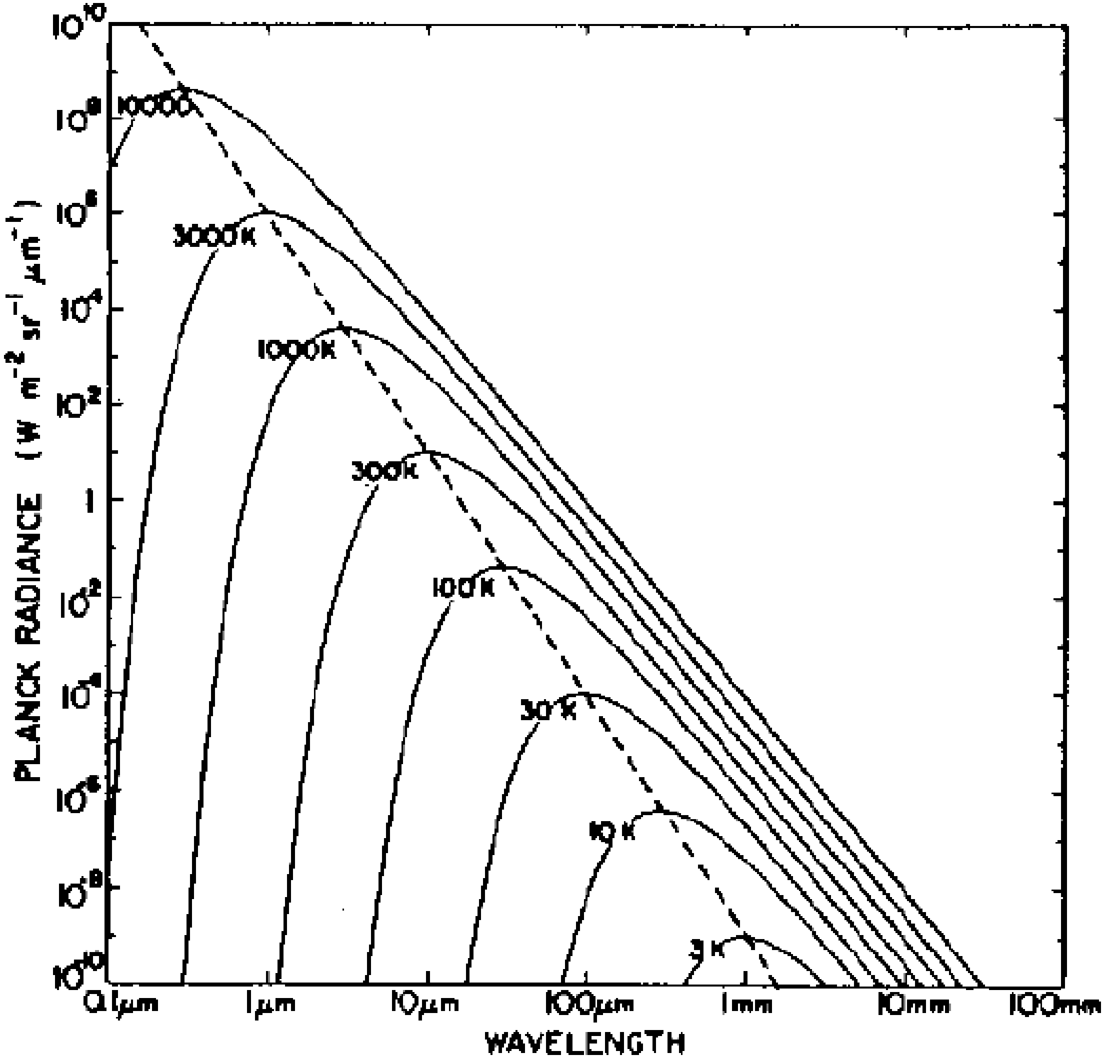
c is the velocity of light; and ***T*** is the absolute temperature of a blackbody.

NOTE: The relations between $B_{\tilde{\nu}}(T)$; $B_\nu(T)$ and $B_\lambda(T)$ are derived using that

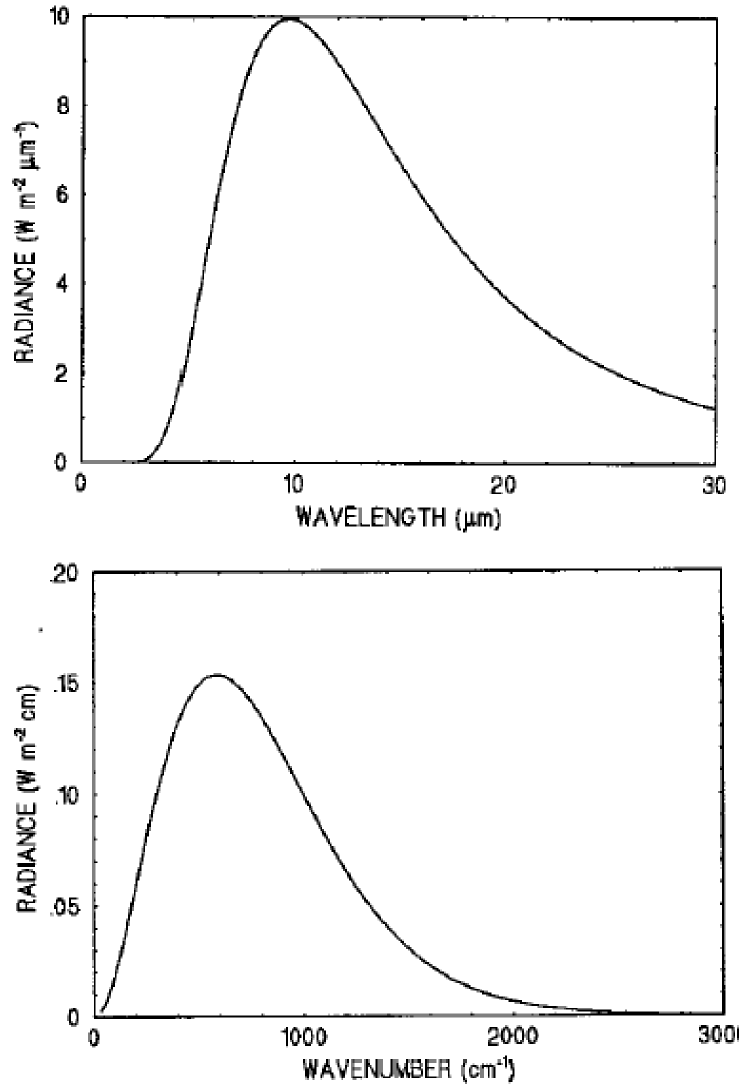
$B_{\tilde{\nu}}(T)d\tilde{\nu} = B_\nu(T)d\nu = B_\lambda(T)d\lambda$, and that $\lambda = c/\tilde{\nu} = 1/\nu$ =>

$$B_{\tilde{\nu}}(T) = \frac{\lambda^2}{c} B_\lambda(T) \text{ and } B_\nu(T) = \lambda^2 B_\lambda(T)$$

Planck functions on log-log plot for many temperatures.



Comparison of Planck function per wavelength and per wavenumber spectral intervals for $T = 300$ K.



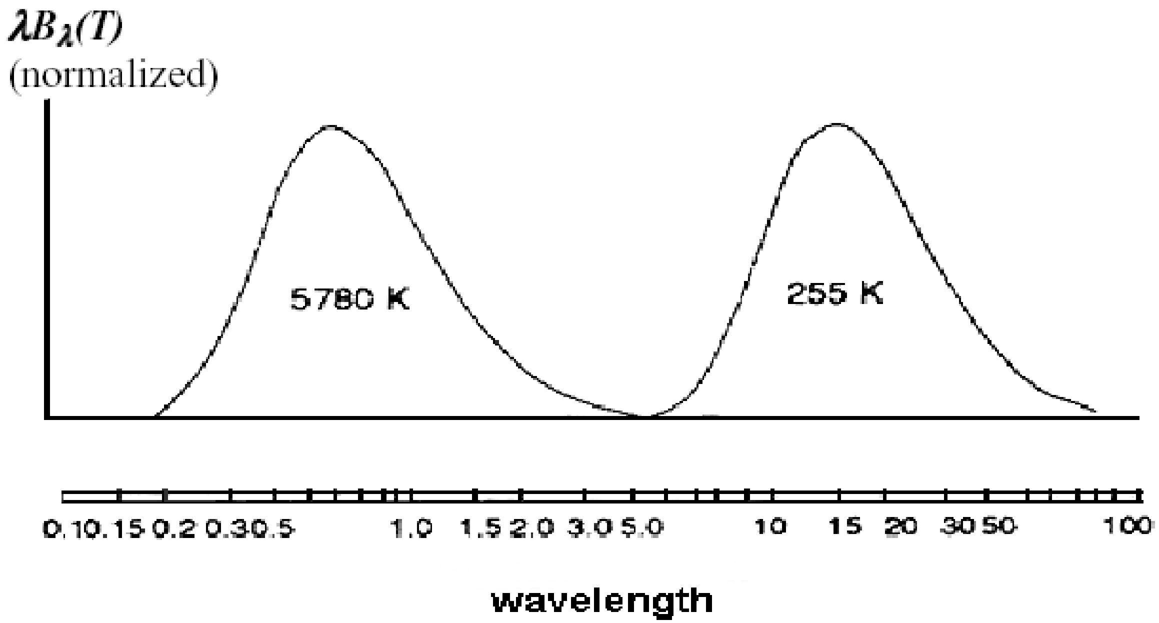


Figure 3.2 Normalized Planck functions for Sun and Earth temperatures

NOTE: Because the sun and earth's spectra have a very small overlap, the radiative transfer processes for solar and infrared regions are often considered as two independent problems.

$$B_\lambda(T) = \frac{2hc^2}{\lambda^5(\exp(hc/k_B T\lambda) - 1)}$$

Asymptotic behavior of Planck function:

- **If $\lambda \to \infty$ (or $\tilde{\nu}$ -> 0) (known as Rayleigh –Jeans distributions):**

$$B_\lambda(T) = \frac{2k_B Tc}{\lambda^4}$$

$$B_{\tilde{\nu}} = \frac{2k_B T\tilde{\nu}^2}{c^2}$$

NOTE: This longwave limit has a direct application to passive **microwave** remote sensing.

- **If λ -> 0 (or $\tilde{\nu} \to \infty$):**

$$B_\lambda(T) = \frac{2hc^2}{\lambda^5}\exp(-hc/\lambda k_B T)$$

$$B_{\tilde{\nu}} = \frac{2h\tilde{\nu}^3}{c^2}\exp(-h\tilde{\nu}/k_B T)$$

Stefan-Boltzmann law.

The **Stefan-Boltzmann law** states that the total power (energy per unit time) emitted by a **blackbody,** per unit surface area of the **blackbody**, varies as the fourth power of the temperature.

$$\boxed{\mathbf{F = \pi\, B(T) = \sigma_b\, T^4}} \qquad [3.4]$$

where σ_b is the *Stefan-Boltzmann constant* ($\sigma_b = 5.671 \times 10^{-8}$ W m^{-2} K^{-4}),

F is radiant flux [W m^{-2}], and **T** is blackbody temperature [K];

$$B(T) = \int_0^{\infty} B_\lambda(T)\, d\lambda$$

Example: Compare the flux emitted by a 290 K surface (e.g. the Earth) and a 5800 K surface (the Sun).

$$F_{\mathrm{Earth}} = (5.67 \times 10^{-8}\ \mathrm{W\ m^{-2}\ K^{-4}})(290\ \mathrm{K})^4 = 401\ \mathrm{W\ m^{-2}}$$

$$F_{\mathrm{Sun}} = (5.67 \times 10^{-8}\ \mathrm{W\ m^{-2}\ K^{-4}})(5800\ \mathrm{K})^4 = 6.42 \times 10^7\ \mathrm{W\ m^{-2}}$$

The ratio of flux emitted is $20^4 = 160,000$!

Spectral Integral of Planck Function

Integrals of the Planck function over spectral bands are needed for radiative flux calculations.
For $x = c_2\nu/T > 1$ the following series convergences rapidly

$$\int_{\nu}^{\infty} B_\nu(T)d\nu \ = c_1 \left(\frac{T}{c_2}\right)^4 \ \sum_{n=1}^{\infty} e^{-nx}\left[\frac{x^3}{n} + \frac{3x^2}{n^2} + \frac{6x}{n^3} + \frac{6}{n^4}\right]$$

For $x = c_2\nu/T < 1$ the following series convergences more rapidly

$$\int_{0}^{\nu} B_\nu(T)d\nu \ = c_1 \left(\frac{T}{c_2}\right)^4 \ \left[\frac{x^3}{3} - \frac{x^4}{8} + \frac{x^5}{60} - \frac{x^7}{5040} + \frac{x^9}{272160} - \cdots\right]$$

Brightness Temperature

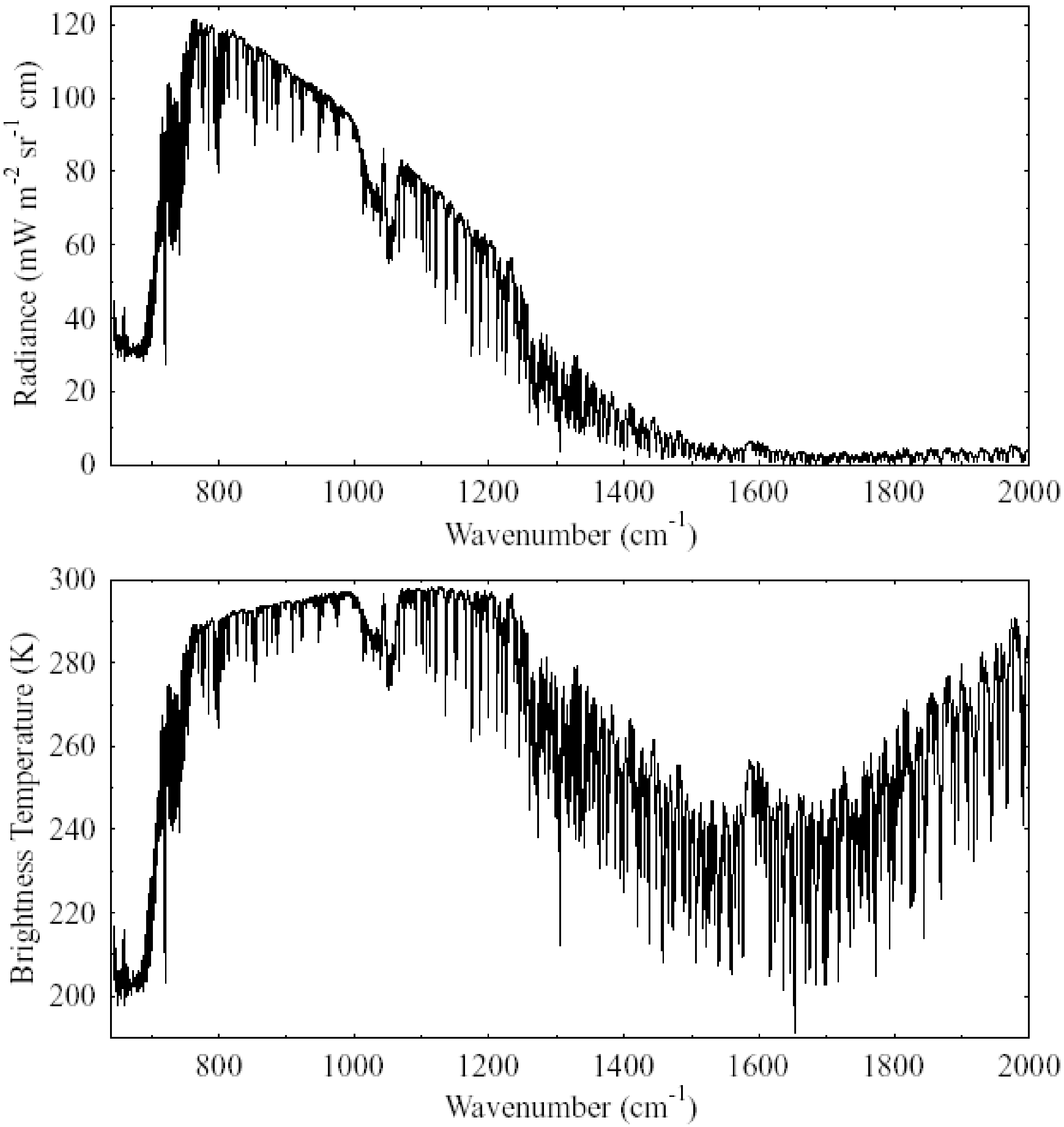
Another way to express radiance - *brightness temperature* is the temperature of blackbody having the same radiance:

$$T_b = \frac{c_2}{\lambda \ln[1 + c_1/(I_\lambda \lambda^5)]}$$

Brightness temperature is very important for remote sensing, as it is often easier to interpret radiance in terms of temperature (see infrared spectrum figure).

An example Earth upwelling infrared spectrum measured by the NAST-I interferometer spectrometer at 54,000 ft (0.25 cm^{-1} resolution). In the atmospheric window region around 10 μm the brightness temperature corresponds reasonable closely to the Earth's surface temperature.

NAST-I Spectrum (CLAMS 2001/7/26 15:29:57)



Wien's displacement law.

The **Wien's displacement law** states that the wavelength at which the blackbody emission spectrum is most intense varies inversely with the blackbody's temperature. The constant of proportionality is Wien's constant (2897 K μm):

$$\boxed{\lambda_m = 2897 / T} \qquad [3.5]$$

where λ_m is the wavelength (in micrometers, μm) at which the peak emission intensity occurs, and **T** is the temperature of the blackbody (in degrees Kelvin, K).

NOTE: this law is simply derived from $dB_\lambda/d\lambda = 0$

NOTE: Easy to remember statement of the Wien's displacement law:

the hotter the object the shorter the wavelengths of the maximum intensity emitted

Example: Compare the wavelength of maximum emission from the Sun (assume a blackbody of 5800 K) and a typical Earth surface (a blackbody at 290 K).

$$\lambda_{\text{max,Sun}} = \frac{2900\ \mu\text{m K}}{5800\ \text{K}} = 0.5\ \mu\text{m} \quad \text{mid visible}$$

$$\lambda_{\text{max,Earth}} = \frac{2900\ \mu\text{m K}}{290\ \text{K}} = 10\ \mu\text{m} \quad \text{mid infrared}$$

Local Thermodynamic Equilibrium

In local thermodynamic equilibrium (LTE): **emission depends only on temperature and absorption properties of matter, not on the radiation field itself.**

In LTE:

- The time between quantum transitions from collisions $<<$ time between transitions from radiation.
- Boltzmann distributions apply to relevant atomic/molecular levels.
- Applies up to 50 to 80 km in Earth's atmosphere (depends on ν).

LTE applies in the Earth's troposphere and stratosphere, but not the mesosphere and thermosphere.
Thomas & Stamnes discuss LTE and also consider non-LTE, where the radiation field and population of quantum states are coupled. However, we will always assume LTE!

3.Emissivity of the surfaces and the atmosphere.

Emissivity = fraction emitted radiance is to that from a blackbody:
$\epsilon_\lambda = I_\lambda^{\text{emitted}}/B_\lambda$. Emitted radiance is thus

$$I_\lambda = \epsilon_\lambda B_\lambda(T)$$

A blackbody has $\epsilon_\lambda = 1$.

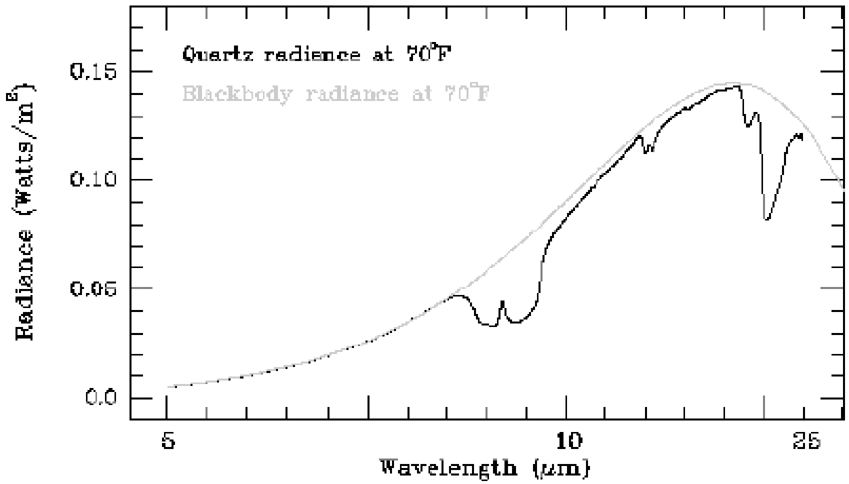
Absorptivity = fraction of incident radiation that is absorbed (a_λ).

- For atmospheric radiation transfer applications, one needs to distinguish between the **emissivity of the surface** (e.g., various types of lands, ice, etc.) and the **emissivity of an atmospheric volume** (consisting of gases, aerosols, and/or clouds).

Emissivity of the surfaces:

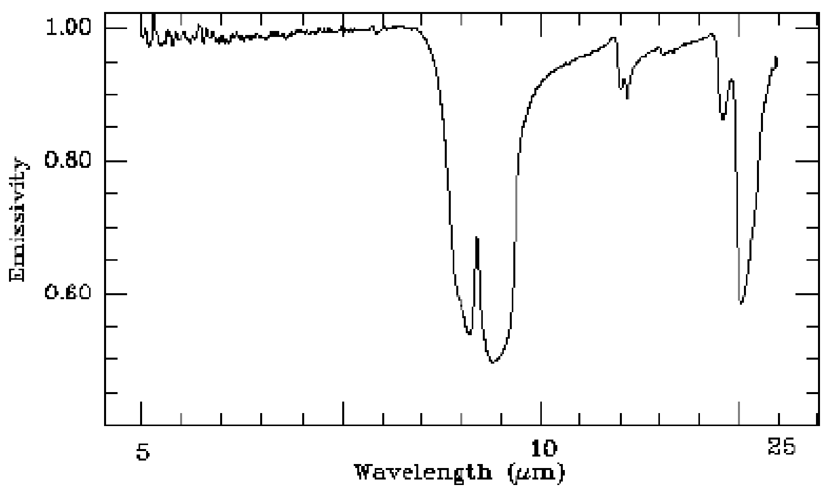
❖ In general, emissivity depends on the direction of emission, surface temperature, wavelength and some physical properties of the surface (e.g., the refractive index).

❖ In thermal IR ($\lambda > 4$ μm), nearly all surfaces are efficient emitters with the emissivity > 0.8 and their emissivity does not depend on the direction. Therefore, the intensity emitted from a unit area surface at a given wavelength is $\mathbf{I_\lambda = \varepsilon_\lambda B_\lambda(T_s)}$



Quartz radiance spectrum along with a blackbody radiance spectrum at the same temperature.

Emitted radiance spectra for a black body and for a quartz surface.



Quartz emissivity spectrum: the result of dividing quartz radiance by blackbody radiance at the same temperature.

The quartz emissivity spectrum form dividing the two radiances spectra.

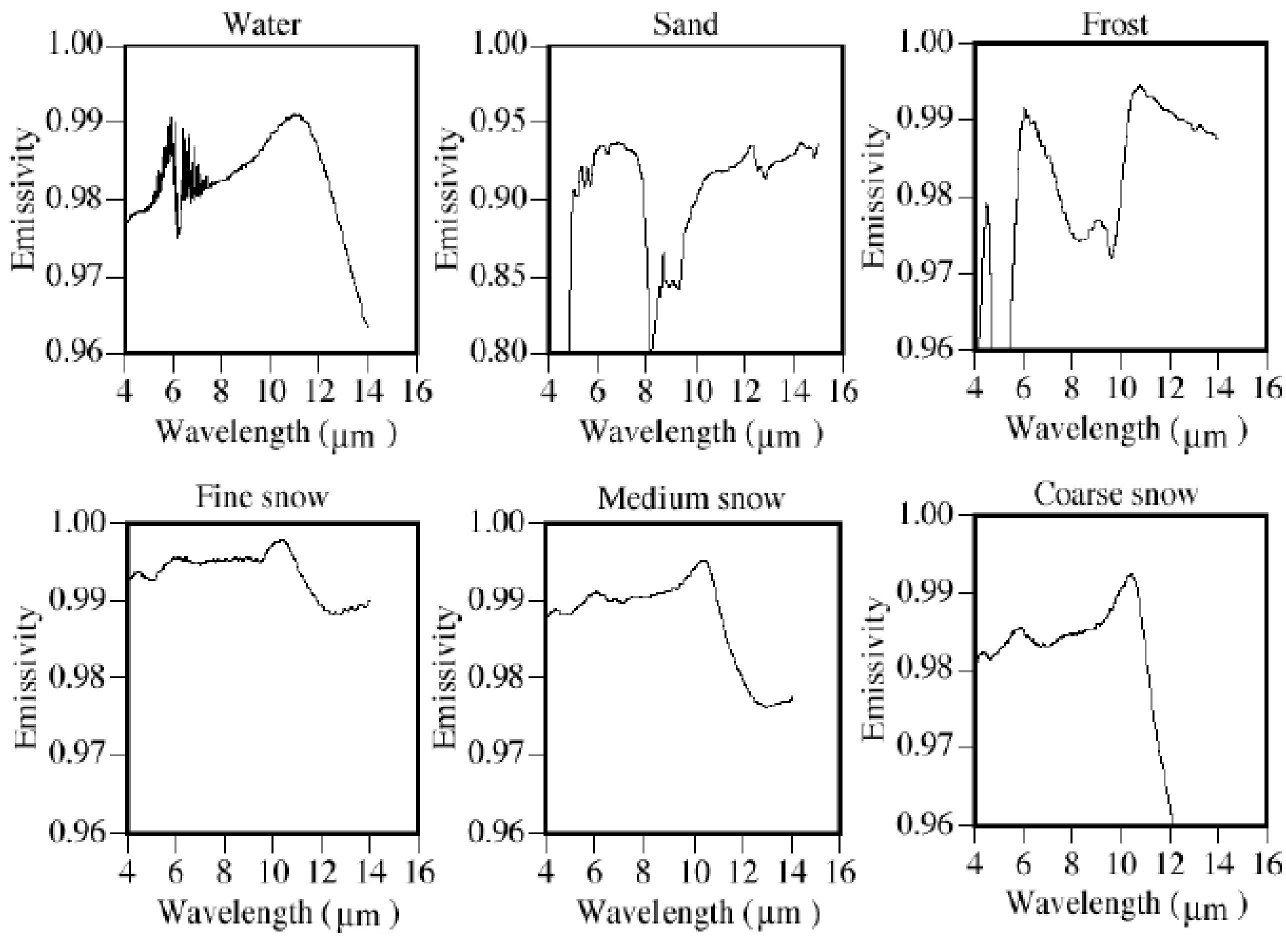


Figure 3.3 Examples of spectral emissivity of some surfaces.

Table 3.1 Average emissivities of some surfaces in the IR region from 10-to12 μm.

Surface	Emissivity
Water	**0.993-0.998**
Ice	**0.98**
Green grass	**0.975-0.986**
Sand	**0.949-0.962**
Snow	**0.969-0.997**
Granite	**0.898**

Kirchhoff's law.

The **Kirchhoff's law** states that the emissivity, ε_λ, of a medium is equal to the absorptivity, A_λ, of this medium under thermodynamic equilibrium:

$$\boxed{\varepsilon_\lambda = A_\lambda} \qquad [3.6]$$

where ε_λ is defined as the ratio of the emitting intensity to the Planck function;

A_λ is defined as the ratio of the absorbed intensity to the Planck function.

For a **blackbody**: $\varepsilon_\lambda = A_\lambda = 1$ For a **non-blackbody**: $\varepsilon_\lambda = A_\lambda < 1$

For a **gray body**: $\varepsilon = A < 1$ (i.e., no dependency on the wavelength)

NOTE: **Kirchhoff's law** applies to gases, liquids and solids if they in TE or LTE.

Emissivity of the atmospheric volume:

Absorption and thermal emission of the atmosphere volume is **isotropic**.

Kirchhoff's law applied to volume thermal emission gives

$$j_{\lambda,thermal} = \beta_{a,\lambda} B_\lambda(T) \quad [3.7]$$

where $\beta_{a,\lambda}$ is the absorption coefficient of the atmospheric volume and

j_λ is the **thermal emission coefficient** which relates to **the source function** J_λ

(introduced in Lecture 2) as $J_\lambda = (j_{\lambda,\, thermal} + j_{\lambda,\, scattering}) / \beta_{e,\lambda}$

and $\beta_{e,\lambda}$ is the extinction coefficient of the atmospheric volume.

Recall the elementary solution of radiative transfer equation (Eq.[2.16] derived in Lecture 2):

$$I_\lambda(s_1) = I_\lambda(0)\exp(-\tau_\lambda(s_1;0)) + \int_0^{s_1} \exp(-\tau_\lambda(s_1;s)) J_\lambda \beta_{e,\lambda} ds$$

- For a **non-scattering medium in the thermodynamical equilibrium:**

$J_\lambda = B_\lambda(T)$, where $B_\lambda(T)$ is Plank's function.

Also, for non-scattering media, we have $\beta_{e,\lambda} = \beta_{a,\lambda} = k_\lambda \rho$, where k_λ is the mass absorption coefficient and ρ is the density (see Lecture 2).

Thus, the **solution** of the **equation radiative transfer** for this case is [3.8]

$$\boxed{I_\lambda(s_1) = I_\lambda(0)\exp(-\tau_\lambda(s_1;0)) + \int_0^{s_1} \exp(-\tau_\lambda(s_1;s)) B_\lambda(T(s)) k_\lambda \rho ds}$$

NOTE: The optical depth in Eq.[3.8] is due to absorption only, so

$$\tau_\lambda(s_1;s) = \int_s^{s_1} \beta_{e,\lambda}(s) ds = \int_s^{s_1} k_\lambda \rho ds$$